

# TITLE: Fractions, Geometry, & Radial Design

## ESSENTIAL QUESTIONS:

What is the relationship between fractions, geometry and radial designs?

How can we use our understanding of math to identify and create visually aesthetically pleasing radial designs?

**OBJECTIVES:** SWBAT (Students Will Be Able To) Identify examples of radial symmetry in art and nature. SWBAT demonstrate an understanding of radial symmetry and congruent shapes to create their own unique radial designs.

**VOCABULARY:** radius, congruence, fraction, ratio, radial symmetry, rotational symmetry, bilateral symmetry, asymmetry, order of symmetry, line of symmetry, balance, repetition, pattern, isosceles triangle, right triangle, hypotenuse, and regular geometric shapes

GRADES	ACTIVATING STRATEGIES	MATERIALS	TEACHING & LEARNING ACTIVITIES	COMMON CORE STANDARDS	REFLECTION/ASSESSMENT
<p><b>K-2</b></p>	<ul style="list-style-type: none"> <li>Provide tactile (or visual) examples of objects with radial symmetry for students to hold and examine. For example, gather students in a circle and pass around a daisy, an orange slice, a sea star, a wheel, a compass, a paper snowflake, (or present pictures of these objects) and discuss the similarities among the objects, drawing attention to the similar geometry and congruent shapes.</li> <li>Define radial symmetry and discuss how each object exemplifies this form of symmetry. Be sure to draw attention to the <i>lines of symmetry</i> in each object.</li> <li>Discuss the visually pleasing nature of radial designs and present famous works of art and architecture from around the world that display these designs.</li> <li>Students are often intrigued by the radially symmetrical work of M.C. Escher, a Dutch artist who was famous for his mathematically based designs. Show students examples of his work, such as Circle Limit IV, or introduce them to his work through the whimsical children's book "Palazzo Inverso" by D.B. Johnson. Ask students to identify lines of symmetry in the</li> </ul>	<ul style="list-style-type: none"> <li>coffee filters—one per student</li> <li>spray bottle of water (optional)</li> <li>markers</li> <li>scissors</li> <li>glue</li> <li>square pieces of white background paper approximately the same size as the coffee filters—one per student</li> <li>for the scaled up lesson—one square piece of construction paper per student as well as a square piece of white background paper</li> <li>for the scaled up lesson--quarter circle cardboard template (made by the instructor) for students to</li> </ul>	<p>Begin the teaching and learning activities with a discussion of Mexican Talavera Tiles. The Metropolitan Museum of Art offers a description of Talavera tile that explains that the majority of these eighteenth-century maiolica (my yo leek ah) ceramics were made in Puebla, Mexico and much of it was blue and white in palette. However, students can also consider how Talavera's "stylistic influences can be traced back to a fusion of Spanish, Arabic, Chinese, and Italian origins" (<a href="http://www.fm.coe.uh.edu">http://www.fm.coe.uh.edu</a>).</p> <p>The instructor can show students many pictures of this type of tile and ask the students some questions about the tiles. For example:          What colors do you see here?          What shapes do you see here?          How are the shapes placed/arranged?          Where are the lines of symmetry in these tiles?</p> <p>To begin the art activity, give kindergarten students coffee filters. Show the students how to fold the filters in half and in half again so that the filters have been folded into quarters.          Using a spray bottle, spray the front</p>	<p><b>Art</b>          National Core Arts Standards for Visual Arts - Anchor Standard 1: Generate and conceptualize artistic ideas and work</p> <p>Anchor Standard 1's Essential Question: How does knowing the context history, and traditions of art forms help us create works of art and design?</p> <p>Enduring Understanding: People create and interact with objects, places, and design that define, shape, enhance, and empower their lives.</p> <p><b>Math:</b>          CCSS.Math.Content.K.MD.A.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference.          CCSS.Math.Content.K.G.A.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").          CCSS.Math.Content.K.G.B.4</p>	<p>K: Students should be assessed on whether they were able to accurately use at least one line of symmetry (out of two) to create a Talavera tile. This can be done when the coffee filter is folded and colored or after the folds arte cut. Students are identifying the line of symmetry when folding the filter and when cutting along the fold. Students should express that the filter is being divided into equal parts, and should be able to identify halves of the filter after the first cut. Students should also be assessed on whether they were able to use the required number of colors as determined by the teacher. This is an opportunity to assess the students' ability to identify colors as well.</p> <p>1: Students should be assessed on whether they were able to accurately use two lines of symmetry to create a Talavera tile. This can be done when the coffee filter is folded and colored or after the folds arte cut. Students are identifying the line of symmetry when folding the filter and when cutting along the fold. Students should then be asked what fraction the line or lines of symmetry create after the cut (1/2s and 1/4ths). Students can also be assessed on choosing colors within a color palette (i.e. warm or cold, etc.) and whether they can spell the colors they create (ELA</p>

	<p>book and in Escher's work.</p> <ul style="list-style-type: none"> <li>As an extension activity, pass around images that <i>do</i> or <i>do not</i> display radial symmetry and ask students to work in groups to discern which images display radial symmetry. To scale this up for more advanced students, include image that display bilateral symmetry and ask students to separate images into three groups—<i>asymmetrical</i>, <i>bilaterally symmetrical</i>, and <i>radially symmetrical</i>.</li> <li>This can be played as a timed game, or done as a group activity with students sharing their results out to the class at the end, followed by class discussion of the results. Students can also sketch images of the objects and draw lines of symmetry on top of the images or students can draw directly onto printed out images of objects that display radial symmetry.</li> <li>As another extension activity, ask students to use manipulatives to create radially symmetrical designs.</li> <li>Pass around kaleidoscopes, or even make kaleidoscopes using cardboard paper towel tubes, tinfoil, and colored paper. Discuss with the students how the designs inside the kaleidoscope are <b>symmetrical</b> and <b>radiate</b> from the center of the design.</li> <li>Show students the website <a href="http://weavesilk.com">weavesilk.com</a> which is a website that allows users to create colorful digital images on a black background.</li> <li>Demonstrate how students can create their own radially symmetrical design by choosing the degree of rotational symmetry from none to 6 degrees.</li> <li>Students can save the images they create and print them out to display or show their classmates.</li> <li>As students begin to explore radial symmetry through your visual or tactile examples, you will provide an explanation of radial symmetry that is tailored for your particular classroom and age group. You</li> </ul>	<p>trace a curve on their paper</p>	<p>and back of each filter with water so that the filter is slightly damp and can be spread flat. Ask the students to use markers to draw their designs on the damp filters. Students may use only blue markers or they may use a variety of colors common to the tiles. Students can open the filter to view their design. They should allow the filters to dry before cutting them into fourths along the folds. Draw students' attention to the fractions created by folding the paper. Students can then glue these filters on a white square background piece of paper such that each fourth of the filter fits in the corner of the background paper and touches along the edge.</p> <p>To scale this lesson up for older or more advanced learners, students can use a square piece of construction paper. Students can fold the construction paper into fourths and use a quarter circle cardboard template to trace a curve on their paper. Draw students attention to the fractions created by folding the paper. Students can use scissors to cut along this curve so that when they open their paper, the paper is now a circle instead of a square. Students should be shown how to hold the paper so that the 2 folds are on the bottom and 1 fold is on the left <i>before</i> they trace and cut their paper. Students can then use crayons or markers to draw symmetrical designs on their circle. Finally, students can cut along the folds of their paper and glue their cut circle fourths on a white square background piece of paper such that each fourth of the circle fits in the corner of the background paper and touches along the edge. This adaption allows more advanced learners to practice fine motor skills through the more difficult cutting and tracing and it also requires students to <b>create their own</b> symmetrical design rather than relying on the marker or crayon to bleed through the paper. Students who require further extension can add their own</p>	<p>Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).</p> <p>CCSS.Math.Content.1.G.A.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.</p> <p>CCSS.Math.Content.1.G.A.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.</p> <p>CCSS.Math.Content.2.G.A.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.</p>	<p>activity).</p> <p>2: Students should be assessed on whether they were able to accurately use two lines of symmetry to create a Talavera tile. The lines of symmetry at this level should also intersect at the true center of the circle. This can be done when the coffee filter is folded and colored or after the folds are cut. Students are identifying the line of symmetry when folding the filter and when cutting along the fold. Students at this level should be able to identify that two of the four pieces of the filter are equal to one half of the filter to build knowledge of fraction equivalency. Students should also be asked to identify which parts of their design are symmetrical by pointing to features in their design that are mirrored in the other quarters of their filter after coloring.</p> <p><i>*Be sure to explore and reevaluate the answers to the essential questions throughout the activity!</i></p>
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	<p>may want to review certain vocabulary as needed.</p> <ul style="list-style-type: none"> <li>You may want to ask some of the following questions:</li> <li>What does it mean if an image or object is symmetrical?</li> <li>What are the different <i>types</i> of symmetry and how are they different?</li> </ul>		<p>symmetrical design to the center of the white background paper.</p>		
<p><b>3-4</b></p>	<ul style="list-style-type: none"> <li>Where is their <b>line of symmetry</b> in a symmetrical image?</li> <li>Where do we see symmetry in the world around us? (For example, a butterfly displays bilateral symmetry while a bicycle wheel displays radial (or rotational) symmetry.)</li> <li>Where else do we see examples of parts to a whole? (For example, a slice of pizza is an example of one part of a whole pie.)</li> </ul>	<ul style="list-style-type: none"> <li>two pieces of black construction paper</li> <li>scissors</li> <li>circle tracer or a paper plate to trace a large circle</li> <li>tissue paper</li> <li>glue</li> <li>pencil to draw designs</li> </ul>	<p>For 3-4<sup>th</sup> grade students, begin the teaching and learning activities with a discussion of gothic architecture. A great way to do this is by reading to students excerpts from the book, "Cathedral: The Story of Its Construction" by David Macaulay. It tells the story of the cathedral construction, the parts of a cathedral, and how these magnificent structures were planned and built. Additionally, Macaulay's PBS special about cathedrals can be found easily on youtube and is an excellent resource for teachers who wish to show students more about the construction of gothic structures.</p> <p>Show students several examples of gothic rose windows and ask them to compare and contrast the windows.</p> <p>Ask students: What shapes and colors do you see? Where do these shapes and colors repeat? Can you explain how this window displays a pattern?</p> <p>Note that some windows exhibit decorative images while other windows display more complex stories and biblical themes.</p> <p>Draw students' attention to the designs of the windows and encourage them to locate the lines of symmetry along which these radially symmetrical designs repeat themselves.</p> <p>For 3<sup>rd</sup> grade students, give students two pieces of black construction paper and scissors. Students can use a circle tracer or a paper plate to trace a large circle onto one of their pieces of black paper. Students should then stack the pieces of black paper and cut the circle out so that they cut both circles at the same time.</p>	<p><b>Art</b> Anchor Standard 1: Generate and conceptualize artistic ideas and work</p> <p>Anchor Standard 1's Essential Question: How does knowing the context history, and traditions of art forms help us create works of art and design?</p> <p>Enduring Understanding: People create and interact with objects, places, and design that define, shape, enhance, and empower their lives.</p> <p><b>Math:</b></p> <p>CCSS.Math.Content.3.NF.A.1 Understand a fraction <math>1/b</math> as the quantity formed by 1 part when a whole is partitioned into <math>b</math> equal parts; understand a fraction <math>a/b</math> as the quantity formed by <math>a</math> parts of size <math>1/b</math>.</p> <p>CCSS.Math.Content.3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</p> <p>CCSS.Math.Content.3.NF.A.3.a Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.</p> <p>CCSS.Math.Content.3.NF.A.3.b Recognize and generate simple equivalent fractions, e.g., <math>1/2 = 2/4</math>, <math>4/6 = 2/3</math>. Explain why the fractions are equivalent, e.g., by using a visual fraction model.</p> <p>CCSS.Math.Content.3.NF.A.3.c Express whole numbers as fractions, and recognize fractions</p>	<p>3: Students should be assessed on whether they can identify the equal parts of their circular rose windows. Can they identify the various fractions of the whole they created? (<math>1/2</math>s, <math>1/4</math>ths, <math>1/8</math>ths?) Can they identify the equivalencies of the pieces of the rose window (<math>1/2 = 1/4 + 1/4</math>)? Can they identify, classify and define the shapes they used in the design of their rose window? Can they explain the equivalency in area of equivalent fractions within the rose window?</p> <p>4: Use the assessments for 3<sup>rd</sup> grade, but in addition assess whether students can identify the many lines of symmetry in their rose windows.</p> <p><i>*Be sure to explore and reevaluate the answers to the essential questions throughout the activity!</i></p>

			<p>Students can then fold one of the circles in half, fold again to create fourths, and fold one more time to create eighths. Draw students attention to the fractions created by folding the paper. Students should be directed to cut relatively small shapes into the folded edge of the paper, leaving about a half of an inch in between each cut so that they have a large enough area of black paper on which to glue the tissue paper.</p> <p>When they are finished cutting, students can fold their second circle into eighths and overlay their first circle so that they can trace and then cut the same shapes. Next, students can glue tissue paper to the back of one of the black circles so that the colors of the paper create a radial design. Therefore, each color should be arranged so that the final product is radially symmetrical.</p> <p>Students can then glue the two pieces of black paper together to finish the window design.</p> <p>To scale this lesson up for 4<sup>th</sup> graders, students can be required to incorporate specific geometric shapes such as diamonds, triangles, circles, etc. into their designs.</p>	<p>that are equivalent to whole numbers. Examples: Express 3 in the form <math>3 = 3/1</math>; recognize that <math>6/1 = 6</math>; locate <math>4/4</math> and 1 at the same point of a number line diagram.</p> <p>CCSS.Math.Content.3.NF.A.3.d Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>, and justify the conclusions, e.g., by using a visual fraction model.</p> <p>CCSS.Math.Content.3.G.A.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.</p> <p>CCSS.Math.Content.3.G.A.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as <math>1/4</math> of the area of the shape.</p> <p>CCSS.Math.Content.4.NF.A.1 Explain why a fraction <math>a/b</math> is equivalent to a fraction <math>(n \times a)/(n \times b)</math> by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</p> <p>CCSS.Math.Content.4.G.A.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the</p>	
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				figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	
<b>5-6</b>		<ul style="list-style-type: none"> <li>▪ square piece of white paper (large is preferred, but printer paper cut to 8"x8" will also work)</li> <li>▪ pencil</li> <li>▪ scrap paper for making carbon paper—if desired to trace designs, otherwise a window can be used or a dark marker can be used to create the letters so that they will show through the paper</li> <li>▪ markers or colored pencils to color the designs created by the lettering</li> </ul>	<p>For 5-6<sup>th</sup> grade students, begin the teaching and learning activities with a discussion of Islamic Art. A great way to do this is by showing students a video regarding Islamic Art from the PBS Learning Media Series. (<a href="https://why.pbslearningmedia.org/resource/islam08.socst.world.glob.islamicart/islamic-art/">https://why.pbslearningmedia.org/resource/islam08.socst.world.glob.islamicart/islamic-art/</a>)</p> <p>The video explains <b>basic tenets</b> of Islamic art and faith and shows examples of Islamic art and artifacts. The video draws attention to the <b>prohibition on iconography</b> in Islamic art and focuses on the <b>use of geometric designs and calligraphy</b> to create stunning art. For those teachers wishing to include a Social Studies extension, the PBS series offers a lesson on "Religion and the First Amendment" so that students can understand why religious freedoms are protected in the United States. The BBC also offers an introduction to Islamic art that is very informative and discusses the <b>common geometric motifs</b> seen in Islamic art. (<a href="https://www.bbc.co.uk/religion/religions/islam/art/art_1.shtml">https://www.bbc.co.uk/religion/religions/islam/art/art_1.shtml</a>)</p> <p>Show students several examples of Islamic mosaics and tile design and ask them questions about the art. For example, ask students:      What colors do you see?      What geometric shapes do you see?      Where do you see patterns and which elements repeat themselves?      Draw students' attention to the use of circular motifs that iterate a belief in the infinite nature of Allah and "the famous concept of the</p>	<p><b>Art</b>      Anchor Standard 1: Generate and conceptualize artistic ideas and work</p> <p>Anchor Standard 1's Essential Question: How does knowing the context history, and traditions of art forms help us create works of art and design?</p> <p>Enduring Understanding: People create and interact with objects, places, and design that define, shape, enhance, and empower their lives.</p> <p><b>Math:</b>      CCSS.Math.Content.5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.</p> <p>CCSS.Math.Content.5.G.B.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</p> <p>CCSS.Math.Content.5.G.B.4 Classify two-dimensional figures in a hierarchy based on properties.</p> <p>CCSS.Math.Content.6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.</p> <p>CCSS.Math.Content.6.RP.A.3.c</p>	<p>5: Students should be assessed as to whether they can write an equation that explains the creation of their square's fractional parts (Ex: <math>1/2 = 1/8 + 1/8 = 1/8 + 1/8</math>, or <math>1/2 = 1/4 + 1/8 + 1/8</math>) Exploring the relationship between the real-life fractional parts of the square and the abstract representation of this concept with the equation makes for a good discussion opportunity where students can be assessed. Additionally, students should be assessed on whether they used, identified, accurately created and/or classified the 2-D shapes they are studying in class. The teacher may be able to assign shapes that must be used in the design as an assessment opportunity.</p> <p>6: Students should be assessed similarly as to fifth grade; however the focus should be on identifying the ratio and percentages of pieces of the square to the whole of the square. Additionally, students should be assessed on how they were able to use the coordinate plane transparency to achieve symmetry. Students should be able to recognize that by changing the polarity of a point, they can create symmetry in the four quadrants. This can be scaled up for 1/8ths or down for halves.</p> <p><i>*Be sure to explore and reevaluate the answers to the essential questions throughout the activity!</i></p>

		<p>arabesque, which is defined as 'ornamental work used for flat surfaces consisting of interlacing geometrical patterns of polygons, circles, and interlocked lines and curves'" (Chambers Science and Technology Dictionary 1991).</p> <p>Also show students examples of "Girih' designs" in which five "tessellating polygons of multiple shapes ... are often overlaid with a zigzag network of lines" (<a href="https://kidworldcitizen.org/islamic-art-lesson-for-kids/">https://kidworldcitizen.org/islamic-art-lesson-for-kids/</a>).</p> <p>Next, show students examples of Islamic calligraphy and present resources regarding the different calligraphic styles (<a href="http://islamicart.com/main/calligraphy/origins.html">http://islamicart.com/main/calligraphy/origins.html</a>).</p> <p>Discuss the importance of calligraphy in Islamic art and show students animated images of letter-writing in Arabic (<a href="https://thearabiclearner.com/the-Arabic-alphabet/">https://thearabiclearner.com/the-Arabic-alphabet/</a>).</p> <p>For 5-6<sup>th</sup> grade students, begin the art activity by giving each student a square piece of paper. Larger paper is preferred, but squares cut from standard printer paper will work as well.</p> <p>Ask the students to fold the paper in half and then in half again to create fourths.</p> <p>Then, ask students to open the paper and fold diagonally so that the paper is folded into eights and the paper is folded into a triangle. Draw students attention to the fractions created by folding the paper.</p> <p>Next, ask the students to write their name on the triangle along the hypotenuse using thick letters like block or bubble. Students can also create their letters using geometric shapes they are studying in class. The letters should fill the space from the top to the bottom of the triangle and students should consider the calligraphic examples they saw to inspire the creation of their own lettering.</p> <p>Next, students should use carbon paper to transfer their lettering to the other triangles.</p>	<p>Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.</p> <p>CCSS.Math.Content.6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.</p>	
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