# **TITLE:** The Golden Ratio & Art

**ESSENTIAL QUESTIONS:**
What is the relationship between mathematical ratios and visually pleasing images?
How can we use our understanding of math to identify and create visually and aesthetically pleasing works of art?

**OBJECTIVES:** SWBAT (Students Will Be Able To) identify examples of the Golden Ratio in art and nature. SWBAT use the golden ratio to create artworks that are inspired by the works of famous artists.

**VOCABULARY:** ratio, golden ratio, fraction, sequence, Fibonacci sequence, pattern, circle, square, concentric, color, color scheme

| GRADES | ACTIVATING STRATEGIES | MATERIALS | TEACHING & LEARNING ACTIVITIES | COMMON CORE STANDARDS | REFLECTION/ASSESSMENT |
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| **K-2** | ▪ Provide tactile (or visual) examples of objects with the golden ratio for students to hold and examine. For example, gather students in a circle and pass around a rose, a nautilus shell, a sunflower, a fern, (or present pictures of these objects) and discuss the similarities among the objects, drawing attention to the similar curvature and scaling. Show students pictures of the golden ratio and discuss how each object exemplifies this ratio. Discuss the visually pleasing nature of this ratio and present famous works of art and architecture that exhibit this ratio.<br>▪ As an extension, pass around images that *do* or *do not* display the golden ratio and ask students to work in groups to discern which images display the ratio. This can be played as a timed game, or done as a group activity with students sharing their results out to the class at the end, followed by class discussion of the results. Students can also sketch images of the objects and draw the golden spiral on top of the images *or* students can draw directly onto printed out images of objects that display the golden ratio. | ▪ Pre-printed page template of the Golden Ratio Circles for each student (done on printer paper if coloring or cardstock if painting)<br>▪ 8x8 inch "background" square paper on which students can paste their circles (any color)<br>▪ Pencils<br>▪ scissors<br>▪ glue/adhesive<br>▪ crayons/colored pencils if using printer paper *or* markers/paint if using cardstock | For K-2nd grade students, begin the teaching and learning activities with a discussion of Vasily Kandinsky.<br>A great way to do this is by reading the students the book, "The Noisy Paint Box" by Barb Rosenstock. It tells the story of the famous abstract painter, Vasily Kandinsky, in a fun and creative way so that K-2 students can understand and enjoy his work.<br>After reading "The Noisy Paint Box", show the students examples of Kandinsky's work, particularly his "Color Study. Squares with Concentric Circles".<br>Ask the students some questions about Kandinsky's work.<br>For example:<br>What colors do you see here? What shapes do you see here? How are the shapes placed/arranged? Where do you look first when you look at this painting & Why? | CCSS.MATH.CONTENT.K.CC.B.4.C Understand that each successive number name refers to a quantity that is one larger.<br>CCSS.MATH.CONTENT.K.MD.A.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference.<br>CCSS.MATH.CONTENT.K.G.B.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.<br>CCSS.MATH.CONTENT.1.MD.A.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.<br>CCSS.MATH.CONTENT.2.MD.A.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.<br>CCSS.MATH.CONTENT.2.MD.A.3 Estimate lengths using units of inches, feet, centimeters, and meters.<br>CCSS.MATH.CONTENT.2.MD.A.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. | K: Students will identify the largest circle in their artwork, then identify which of the diameter measurements' numbers is the largest and smallest.<br><br>1: Students will order their circles from largest to smallest prior to adhering them to the background paper. Students will be able to discuss the relationships of the size of the circles in relation to each other ("This circle is smaller than this circle, but they are both larger than this circle" etc.)<br><br>2: Students will estimate and then measure the diameters of their circles in whole inches. Students will then identify how much larger a circle is when compared to another circle by using a subtraction algorithm of the measurements or by aligning the edges of two overlapping circles to enable them to measure the difference in diameter length. |

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| | ▪ As an outside-of-class extension, give your students a piece of paper with 5 different rectangles printed on it, only one of which exemplifies the Golden Ratio. Each rectangle should be assigned to a letter (A, B, C, D, E). Ask students to poll their friends and family—asking each person which rectangle they think is the most aesthetically pleasing. Students can tally the results (A-E) to record their findings. They can then display these findings in a graph. Students can even share out with the class to combine data and draw conclusions.<br>▪ You may want to ask some of the following questions: What is a ratio? Where do we see or use ratios? (For example, humans are examples of ratios. We have a fairly standard ratio of parts to whole with regards to how long our limbs are, etc.) Where else do we see examples of part to a whole? (For example, a slice of pizza is an example of one part of a whole pie.) Why do we find certain ratios pleasing? (We're not sure, but we see these ratios in nature and humans have copied them in our art and architecture for millennia). | | What questions do you have about this painting?<br>To begin the art activity, give kindergarten students the pre-printed template that comes with this lesson plan (also shown in this slide).<br>The template will show the golden ratio as exhibited through circles.<br>If the template is printed on regular printer paper, students can use colored pencils or crayons to color each circle with the color of their choice.<br>If the template is printed on cardstock, students can use paint or marker to color in the circles.<br>Students will then cut out their circles and arrange them on top of each other in a concentric fashion as Kandinsky did in his "Color Study".<br>Each student should be given a (roughly 8 inch x 8 inch) square piece of paper to color or paint and use as a background.<br>To scale this lesson up for older or more advanced learners, students can be given a template without circles and then they can use circle tracers provided by the teacher (preferably on cardstock) to match up the circle to the corresponding square and trace the circles.<br>This will extend the activity so that students must identify like-sized shapes and use fine motor skills to trace the circles.<br>To scale the lesson further, students could be asked to color their circles using a color scheme—for example: color the circles with only warm colors or with only cool colors. This is an excellent opportunity to collaborate with an art instructor and will require | | |

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| | | | students to demonstrate their understanding of color schemes. | | |
| **3-4** | | ▪ paper rectangles pre-cut to 8" by 13" (can be cut from 11" by 17" ledger/tabloid paper. A template has been provided)<br>▪ Pencils<br>▪ USCS/Inch Rulers<br>▪ scissors<br>▪ coloring materials: crayons, colored pencils, markers, paint | For 3-4th grade students, begin the teaching and learning activities with a discussion of Josef Albers's artwork.<br>A great way to do this is by reading the students the book, "An Eye for Color" by Natasha Wing.<br>It tells the story of the famous abstract painter, Josef Albers, and explores his study of color theory.<br>For 3rd grade students, give students the large Golden Ratio template which is to be printed on 11 x 17 inch paper.<br>The students will then cut out and color their squares, or they could trace the square onto cardstock and cut the squares out and paint them.<br>The students can then arrange their squares on top of each other and paste them together.<br>The class of students can glue their completed work to a larger poster board in a grid fashion akin to Albers's work.<br>Ask the students some questions about Albers's work.<br>For example:<br>What colors do you see here? What shapes do you see here? How are the shapes placed/arranged? How are the colors arranged?<br>What part of this painting is your eye drawn to & Why?<br>What questions do you have about this painting? | CCSS.MATH.CONTENT.3.MD.C.7 Relate area to the operations of multiplication and addition.<br>CCSS.MATH.CONTENT.3.MD.C.7.A Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.<br>CCSS.MATH.CONTENT.3.MD.C.7.B Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.<br>CCSS.MATH.CONTENT.3.MD.C.7.D Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.<br>CCSS.MATH.CONTENT.3.MD.D.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.<br>CCSS.MATH.CONTENT.3.G.A.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.<br>CCSS.MATH.CONTENT.3.G.A.2 | 3: Have students find the area of their squares by multiplying the length of one side by itself. Then have them cover the square with square inch tiles to check their answer<br>Students find the dimensions of their square's corresponding "golden rectangle" using the Fibonacci sequence (Ex: A square with 8" sides is part of an 8" by 13" rectangle, a square with 3" sides is part of a 3" by 5" rectangle, etc.) Then students will find the area of the accompanying rectangle using square inch tiles and the area formula for parallelograms. Students then find the area of the "whole" golden rectangle by adding the area of the square and the accompanying rectangle contained within each golden rectangle. (EX: An 8" by 13" rectangle is a golden rectangle which has an area of 104 $in^2$, and is composed of an 8" by 8" square and a 5" by 8" rectangle, also a golden rectangle. The area of the square is 64 $in^2$, the area of the 5" by 8" rectangle is 40 $in^2$, and the area of the entire golden rectangle is 64 $in^2$ plus 40 $in^2$, or 104 $in^2$.)<br>Have students create questions for their classmates about their rectangles and squares. For example, a student may ask the class for a missing dimension of the rectangle but provide the area of the square within the rectangle. |

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| | | | For 4th grade, students can use a ruler and an 8 x 13 inch piece of paper to measure out the squares which should follow the Fibonacci Sequence<br>This means that the first square will be 8 x 8 inches, the next square will be 5 x 5 inches, the following square will be 3 x 3 inches, etc.<br>The instructor should demonstrate this measurement in a step-by-step fashion for the class.<br>After creating their squares, students can color and paste them as the 3rd graders did, *or* the students can consider the work of Frank Stella, an artist who was inspired by the work of Josef Albers.<br>Students can compare Stella's work to Albers's work and note the placement of the squares in a careful concentric arrangement.<br>Ask students to describe how Stella's and Albers's squares are different and how they are similar.<br>Students can place their squares as Stella did which will require them to use their ruler to ensure even measurements.<br>The instructor can demonstrate this for the class.<br>As an extension, students can consider *other* examples of Frank Stella's work in which the squares are broken into triangles that do not all share the same color schemes.<br>Students can consider how the triangles complement each other and the overall artwork.<br>To extend this activity in their own artwork, students cut their squares diagonally (after pasting) to create four triangles. | Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.<br>CCSS.MATH.CONTENT.4.MD.A.3<br>Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.<br>CCSS.MATH.CONTENT.4.G.A.1<br>Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.<br>CCSS.MATH.CONTENT.4.G.A.2<br>Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.<br>CCSS.MATH.CONTENT.4.G.A.3<br>Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. | |

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| | | | Each student can pass two of his/her triangles to the student to his/her right. After passing, students should have two of their own triangles and two of their neighbor's triangles. Students can then paste the triangles together on a square base to create a finished piece. Pieces can be displayed together on a grid. | | |
| **5-6** | | ▪ paper rectangles pre-cut to 8" by 13" (can be cut from 11" by 17" ledger/tabloid paper. A template has been provided)<br>▪ Pencils<br>▪ compasses that can be set to inch increments<br>▪ USCS/Inch Rulers<br>▪ coloring materials: crayons, colored pencils, markers, paint<br>▪ OPTIONAL: cardstock and scissors to make traceable circle templates | For 5-6th grade students, begin the teaching and learning activities with a discussion of Bridget Riley's work.<br>A great way to do this is by reading the students a section about the artist Bridget Riley from the "Tate Kids British Art Activity Book" by James Lambert.<br>Kids can explore the Op Art of Bridget Riley through the book and then instructors can show students other examples of her work.<br>After reading about Bridget Riley, show students some examples of her work and ask them how they feel looking at the artwork.<br>For example, ask students: What colors and shapes do you see?<br>Does anything surprise you about this work?<br>How do you think Riley created these artworks?<br>For 5-6th grade students, use the Golden Ratio Template (shown on the slide) so that students can cut out circles that exhibit the golden ratio.<br>Students should then trace their circle in concentric and overlapping designs<br>Using two colors, students can paint or color in an alternating fashion to create an optically stimulating artwork. | CCSS.MATH.CONTENT.6.RP.A.1<br>Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.<br>CCSS.MATH.CONTENT.6.RP.A.2<br>Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. | Students explain how the diameters of their circles are visual representatives of the golden ratio by proving the circles' diameters follow the formula a + b is to a as a is to b.<br>For example, a circle with a five inch radius and a three inch radius will fit the golden ratio's formula of "a plus b divided by a is equal to a divided by b" because 3 + 5 is 8, and 8/5 ≈ 5/3.<br>Students should extend the Fibonacci sequence to find examples of larger numbers where this ratio becomes increasingly closer to the golden ratio. |

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| | | | As an extension, students can use a compass and a piece of 8 x 13 inch paper to measure and draw out the circles in the golden ratio.<br>They can then cut these circles out and use them as tracing templates for their artwork.<br>Students could also complete this lesson in a different fashion by arranging their circles randomly and then using a black pen or marker to create different patterns in each overlapping part of the circle.<br>This option would require greater fine motor skill, would give the students a great deal of personal choice, and would further explore the concept of pattern; however, this option would require more time and would be more successful as a multi-day project. | | |