| ADRIANNA URBANO | MANDALA MADNESS | SCTC 4385 |
|---|---|---|
| **Grade Band:**<br>$6^{th}$ – $9^{th}$ | **Topics:**<br>Geometry<br>Art | **Lesson #:**<br>3- to 4-Day Unit Plan |

## BRIEF LESSON DESCRIPTION

**Description:**
Your role as a gallerist is to identify subtle differences in artwork and to recognize exceptional pieces. Your specific task as an illusionist is to create artwork that may have varying appearances to different viewers and perceptions. Designing and creating a Mandala is your first step to discovering your illusion capabilities. Through mathematics and research, you will explore, design, and create Mandala artwork.

**Leading Question:**
How can we use mathematics to create intricate art?

## PERFORMANCE EXPECTATIONS

**Students Will Be Able To:**

- Understand the connection between geometric theories and artwork
- Understand and utilize technology to enhance pattern recognition
- Create intricate patterns through application of the discussed geometric theories
- Connect concepts of circles, angles, equilateral shapes and symmetry to the creation of art
- Apply research concepts from the Four-Color Theorem to their creations

**Students Will Understand:**
Mandala artwork is based in geometry concepts, such as concentric circles, symmetry, chords, etc. Through this unit, students will apply these conceptual understandings to draw, design and create their own Mandala. Further, students will explore the Four-Color Theorem and apply what they learn to bring their Mandala to life.

**Key Definitions & Concepts:**

- Concentric circles: two or more circles sharing the same center point
- Radius: the distance from the center of the circle to the arc of the circle
- Diameter: the distance from one edge of the circle to another passing through the center point
- Equilateral: polygon of n sides having the same length
- Equiangular: polygon of n sides whose angles are the same measure
- Regular n-gon: polygon of n sides having both equilateral and equiangular properties

## SPECIFIC LEARNING OUTCOMES

**Standards:**

- CCSS.MATH.CONTENT.7.G.A.2: Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions.
- CCSS.MATH.CONTENT.HSG.C.A.2: Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

## BACKGROUND INFORMATION

**Prior Knowledge:**

- Concepts relevant to: symmetry about the origin, symmetry about each axis, reflections, rotations, and circles
- Calculations utilizing radius, diameter, line segments and triangles

| **Mathematical Practices:** | **Disciplinary Core Ideas:** | **Crosscutting Concepts:** |
|---|---|---|
| • Reason abstractly and quantitatively<br>• Model with mathematics<br>• Attend to precision<br>• Use appropriate tools strategically | • Patterns exhibit relationships that can be extended, described, and generalized.<br>• Geometric relationships can be described, analyzed, and classified based on spatial reasoning and/or visualization. | • Observe and make use of patterns<br>• Identify and utilize structure and function |

**Possible Preconceptions/Misconceptions:**

Students will be able to complete the geometry of this lesson, but it is likely to have different methods to solution from each group. Recommended: run through the triangle (number 1 of the exploration) example as a whole-class using inquiry tactics. This will allow students to take charge of their learning while the teacher keeps solution methods uniform.

On design day, student should have no trouble creating the concentric layers of their chosen shape. Making the piece individual through design can throw the use of the Four-Color Theorem. Recommended: be alert through the beginning of the activity to catch students are re-adjust their design process.

On presentation day, the students should be back in their groups to discuss each of their Mandalas and discuss the research topics.

## LESSON PLAN → 5E (+) MODEL

**Engage:**

Engage the students by providing one tablet per group of 3 to 4 students. The tablet should have software that can execute python codes. These codes produce intricate Mandalas that the students will work in their groups to explain their observations, and to make meaningful connections between the artwork and mathematical concepts.

The purpose of this activity is to get students engaged in a compare and contrast activity that is focused around the geometric concepts of circles, rotations and angles via technology. Teachers will use students' responses to lead into the RAFT of the unit: the students' role as a gallerist learning the basics or creating illusionary artwork.

**Explore:**

Students will work in groups containing 3 to 4 members to complete a worksheet that reviews key geometric concepts necessary for completion of this unit. This guided worksheet contains shape-based Mandalas and includes critical thinking questions that are designed to aid students in creating the conceptual bridges between geometry and art design. They will be engaged in small-group and whole-class discussions about the angles, circles, and equilateral shapes inscribed inside the circular designs provided.

To bridge the concepts of geometry to art, students will be engaged in the *benchmark lesson*: what geometric concepts are critical to designing Mandalas, how to apply those concepts, and actively creating their own Mandala. The concepts necessary for the completion of this unit relate to concentric circles and symmetry. This allows students to make unapparent connections more independently and to take charge of their learning.

**Explain:**

Throughout the exploration, the students will engage in discussions that inquire their understanding and knowledge of the information at-hand. Thus, teachers will be informally asking students to explain all topics and relevant connections throughout the entirety of this unit. The guided worksheet contains sections asking students how they arrived at the answer and asking if they notice any patterns between the problems.

The two-part written assignment is designed to be one of the students' formal explanations of the unit plan. The first part of the written assignment is a letter written by the group that is addressed to the head gallerist. It should contain conceptual explanations of how mathematics can be used to create intricate art, (i.e. the letter should contain their explanations of how geometry concepts can be used to design mandalas, and how the Four-Color Theorem is applicable). The second part of the written assignment is each individual students' Mandala creation.

**Elaborate:**

The final project presentation is designed, in part, so that the students communicate the above information to the class along with showcasing their Mandala creation as a proper gallerist. Concluding the exploration, the students will extend their understanding of geometry and art by conducting research to answer the following: (1) what is the Four-Color Theorem, (2) how is it useful, and give an example, (3) where else is the Four-Color Theorem seen, and give an example, and (4) how technology can enhance the creation of Mandalas. The purpose of having the students answer these questions through research is to give them opportunity to make their presentations unique to their interests within the scope of this unit plan. This portion is the *investigation lesson* and the last section of the summative assessment.

**Evaluate:**

This unit plan is designed with having both informal and formal evaluations throughout its entirety. The informal evaluations will occur on all three or four days and in multiple outlets. Predominantly, the students will be asked leading and open-ended questions throughout the exploration of the unit. This will allow teachers to gauge surface-level student understanding. The guided worksheets provide the students with an outline of steps to understanding the connection between geometric concepts and artistic designs. By surveying the students during completion of these worksheets, teachers will be able to hear any misconceptions or misunderstandings, and address such immediately. Also, teachers will be able to gauge the percentage of students grasping the content by listening to their conversations and explanations within the groups.

The formal evaluation within this unit plan occurs on the final day, and it includes the formal letter and presentation. Throughout the unit plan, the students will have had ample time to grapple with geometry and designing their Mandala, ask questions, answer discussion topics and extend their understanding through research. The purpose of the letter is to have students formally explain what they learned and how geometry concepts underlie certain artistic creations. The purpose of the presentation is to have the students communicate the purposes of the letter and showcase their Mandalas to their peers. Each presentation is expected to be between 15 and 20 minutes long per group. Since there are 3 to 4 members per group, each student is expected to between a three (3) to four (4) minute part within their group's total presentation.

**Enrichment:**

This lesson can be extended by having students generate their own codes that design and create Mandala artwork. This allows students to think about the underlying geometric concepts that enable the creation of Mandalas, and to break down these concepts into executable, logical steps.

**See all attachments on the following pages**

Name: _____ Group #: _____ Date: _____

# IDLE-y Drawing Circles

Directions: Observe the following code and produced picture to answer the following questions.

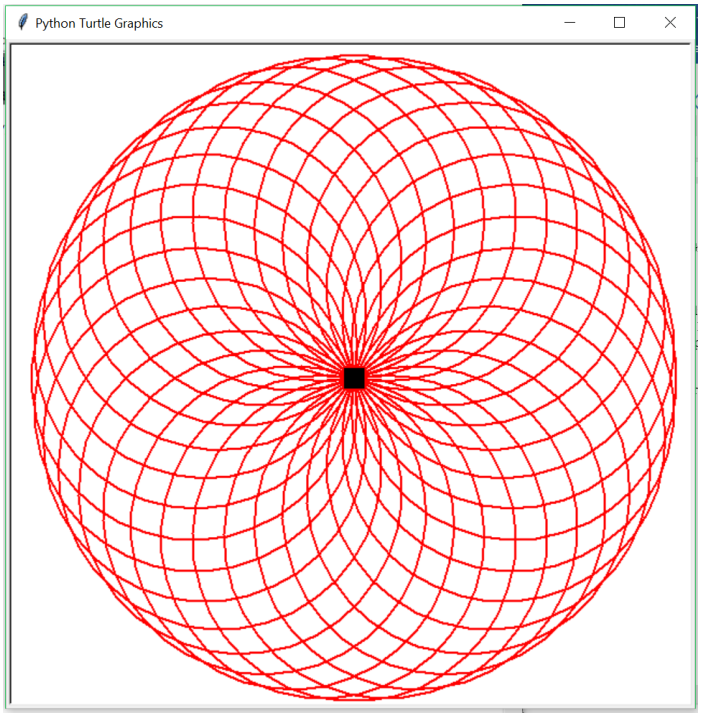
This python code:

```
from turtle import *

angle = 360/32
shape("square")
numOfCircles = 32
pensize(2)
speed(50)

for i in range(numOfCircles):
    rt(angle)
    pencolor("red")
    circle(150)
```

Produces this picture:




1.) Notice the commands: shape("square"), pensize(2), speed(50), pencolor("red") and circle(150). What do you think these commands tell the computer to do? Hint: open IDLE to make changes to these values. Based on your observations, explain the function for each of these commands.

a. shape("square")

b. pensize(2)

c. speed(50)

d. pencolor("red")

e. circle(150)

2.) Notice the lines (1) angle = 360/32 and (2) numOfCircles = 32
    a. What is the significance of 32? How do you know? Hint: discuss your observations by changing '32' in IDLE.

3.) Challenge Questions:
    Notice the line rt(angle). What does this line do? How do you know?

    Without using IDLE, explain what you think lt(angle) would do and why.

Name: _____ANSWER_KEY_____ Group #: _____ Date: _____

IDLE-y Drawing Circles

Directions: Observe the following code and produced picture to answer the following questions.

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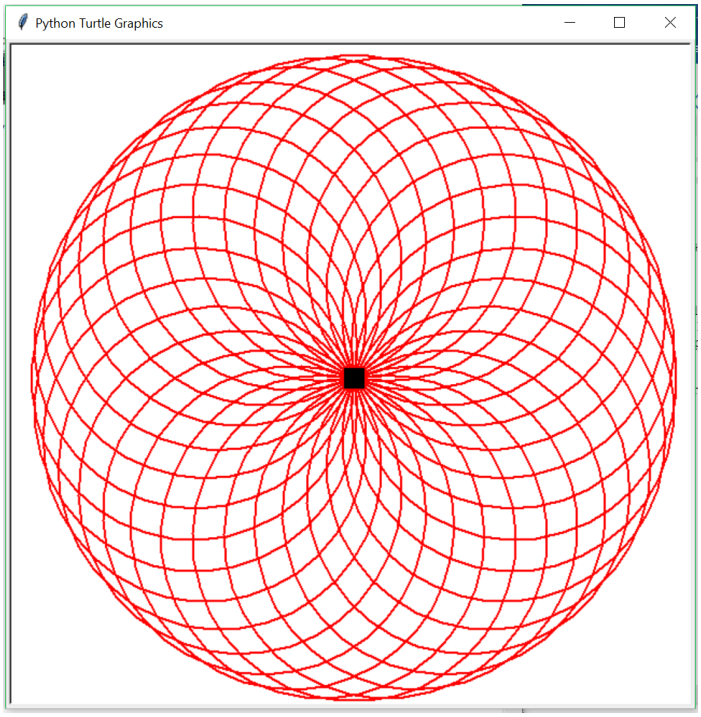
Produces this picture:




1.) Notice the commands: shape("square"), pensize(2), speed(50), pencolor("red") and circle(150). What do you think these commands tell the computer to do? Hint: open IDLE to make changes to these values. Based on your observations, explain the function for each of these commands.

a. shape("square")
   Changes the shape of the pen. Changing the "square" to any other shape changes the appearance of the pen drawing the picture.
b. pensize(2)
   Changes the thickness of the lines drawn. The greater the number, the thicker the line drawn, and vice versa.
c. speed(50)
   Changes the speed at which the pen moves. The greater the number, the faster per second the pen moves, and vice versa.
d. pencolor("red")
   Changes the color of the shape produced. Changing "red" to any other color changes the color of the picture produced.
e. circle(150)
   Changes the size of the circle. The greater the number, the larger the overall picture becomes, and vice versa.

2.) Notice the lines (1) angle = 360/32 and (2) numOfCircles = 32

a. What is the significance of 32? How do you know? Hint: discuss your observations by changing '32' in IDLE.

32 is the determining factor for the angle measure (in degrees). 32 is also the number of circles produced around the center of the outermost circle.

Notice: the center of the outermost circle is considered a pivot point. The angle measure is the number of degrees of rotation about that pivot point.

3.) Challenge Questions:
Notice the line rt(angle). What does this line do? How do you know?

This command tells the computer to turn the pen to the RIGHT the number of degrees determined by ANGLE

Without using IDLE, explain what you think lt(angle) would do and why.

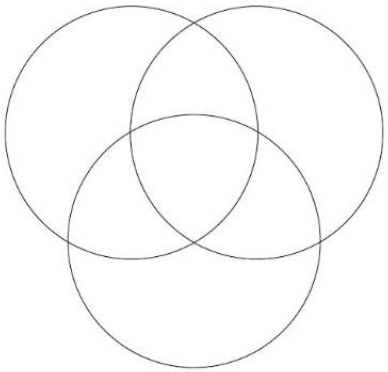
This command tells the computer to turn the pen to the LEFT the number of degrees determined by ANGLE

Name: _____ Group #: _____ Date: _____

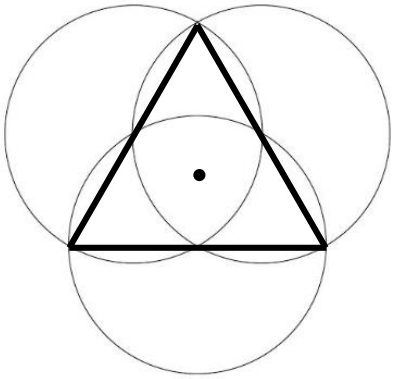
# Mandala Madness

1.) Observe the shapes below to answer the associated questions.

A:



B:



C:

1. What is the difference between A and B?

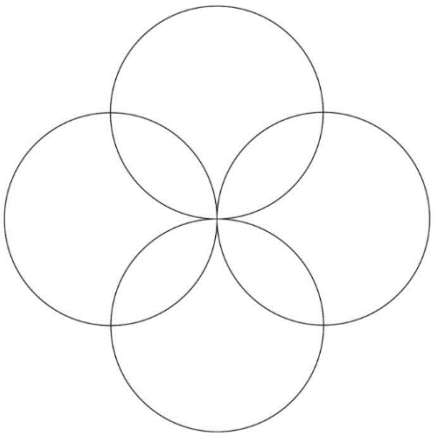
2. Use your protractors and rulers to measure the radius and diameter of the circles, and side lengths of the triangle. Compare these numbers. What do you notice?

3. Challenge:
   If you were to draw a concentric triangle that contained section B, how would you do so?
   a. Explain the process and show all calculations.

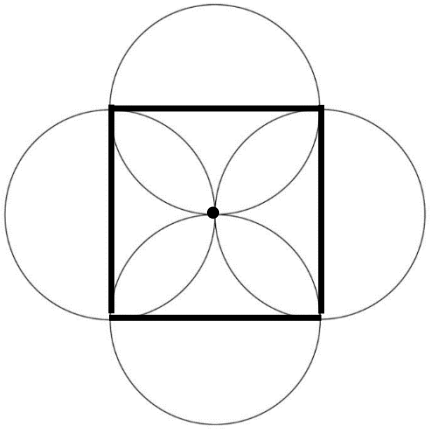
   b. Use the section C to draw the new mandala

2.) Observe the shapes below to answer the associated questions.

A:



B:



C:

1. What is the difference between A and B?

2. Use your protractors and rulers to measure the radius and diameter of the circles, and side lengths of the square. Compare these numbers. What do you notice?

3. Challenge:
   If you were to draw a concentric square that contained section B, how would you do so?
   a. Explain the process and show all calculations.

   b. Use the section C to draw the new mandala

3.) Draw connections between problems 1 and 2.

a. The concentric shapes drawn needed to be rotated to fully contain the total shape from section B in each of the above problems. How many degrees did the shapes have to rotate around the center point? (Show all work: draw the shapes, show calculations, and explain the process when using protractors & rulers).

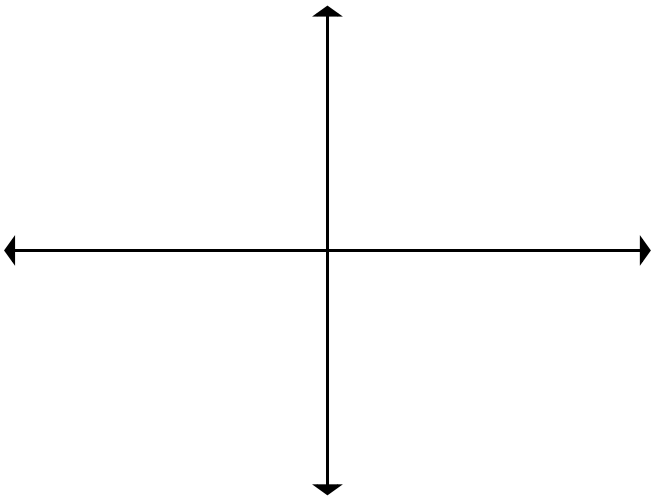
   Triangle:

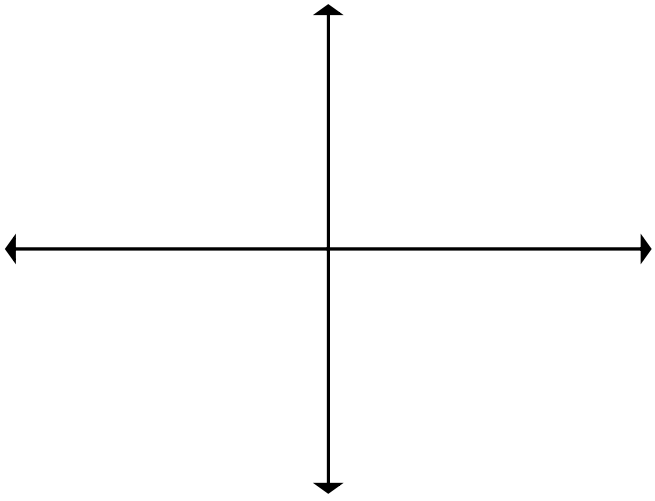
   Square:

b. Notice that all the side lengths of each of the triangles and squares are the same length AND all the angles are the same measure within the triangles and squares respectively. What is the term that defines this property?

   Why is this property significant?

c. Create a three-layered concentric regular triangle and regular square. Notice: problems 1 and 2 are two-layered concentric triangles and squares, respectively.



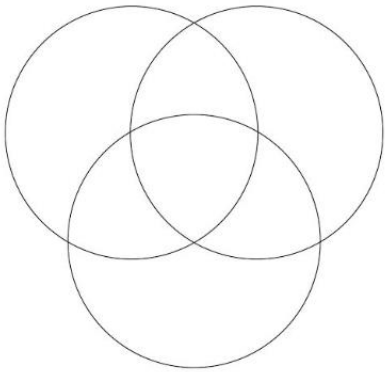


Name: _____ANSWER_KEY_____ Group #: _____ Date: _____

Mandala Madness

2.) Observe the shapes below to answer the associated questions.

A:



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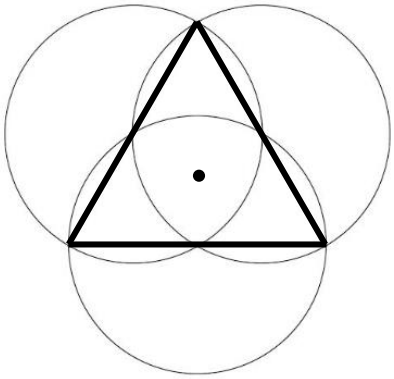
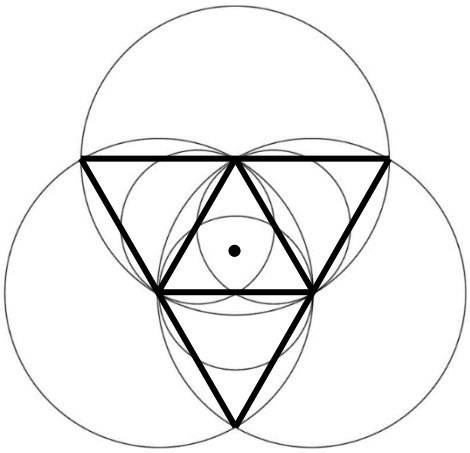


C:



1. What is the difference between A and B?

A triangle is inscribed through the intersections of the circles. The center point of the triangle is the same as the center point of the innermost intersection of the circles.

2. Use your protractors and rulers to measure the radius and diameter of the circles, and side lengths of the triangle. Compare these numbers. What do you notice?

The diameter of each circle is equal to one side length of the triangle. The triangle is equilateral and equiangular.

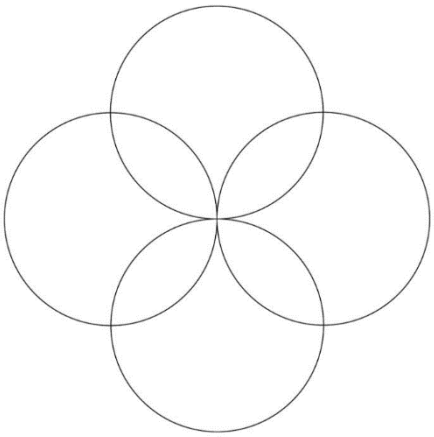
3. Challenge:
   If you were to draw a concentric triangle that contained section B, how would you do so?
   a. Explain the process and show all calculations.

Use properties of equilateral triangles to draw the outer triangle that contains the inner triangle and that shares the same center point. Since the sides have the same length, then the outer triangle side length is double that of the inner triangle. Draw the circles using the diameter of the new side lengths.

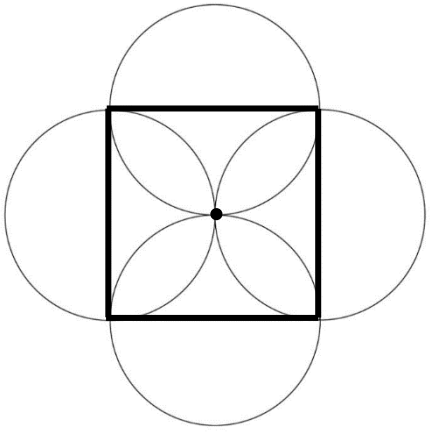
   b. Use the section C to draw the new mandala

3.) Observe the shapes below to answer the associated questions.

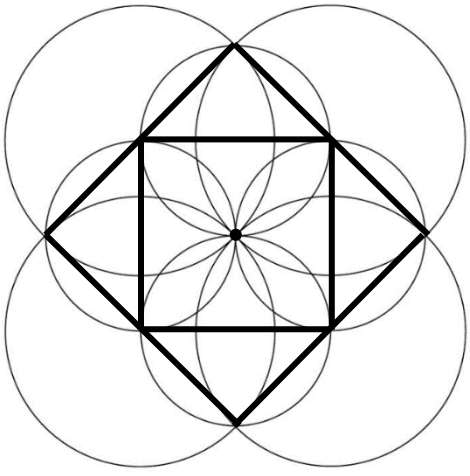
A:



B:



C:



1. What is the difference between A and B?

A square is inscribed through the intersections of the circles. The center point of the triangle is the same as the center point of the innermost intersection of the circles.

2. Use your protractors and rulers to measure the radius and diameter of the circles, and side lengths of the square. Compare these numbers. What do you notice?

The diameter of each circle is equal to one side length of the square. The square is equilateral.

3. Challenge:
   If you were to draw a concentric square that contained section B, how would you do so?
   a. Explain the process and show all calculations.

Divide the square into four equilateral triangles. Follow the process from number 1.

OR

Divide the square into equilateral triangles. Since the sides have the same length, then the outer square side length is double that of the inner square. Draw the circles using the diameter of the new side lengths.

   b. Use the section C to draw the new mandala

4.) Draw connections between problems 1 and 2.

a. The concentric shapes drawn needed to be rotated to fully contain the total shape from section B in each of the above problems. How many degrees did the shapes have to rotate around the center point? (Show all work: draw the shapes, show calculations, and explain the process when using protractors & rulers).

Triangle:

Draw three lines that pass through the center point and through the vertices of the triangle. Using a protractor, measure the angles between the lines (should all be 60 degrees since a regular triangle). Draw three new lines passing through the vertices of the outer-scribed triangle and through the center point. Calculate the degree measure between the old lines and the new using a protractor.

Square:

Draw four lines that pass through the center point and through the vertices of the square. Using a protractor, measure the angles between the lines (should all be 45 degrees since a regular square). Draw four new lines passing through the vertices of the outer-scribed square and through the center point. Calculate the degree measure between the old lines and the new using a protractor.

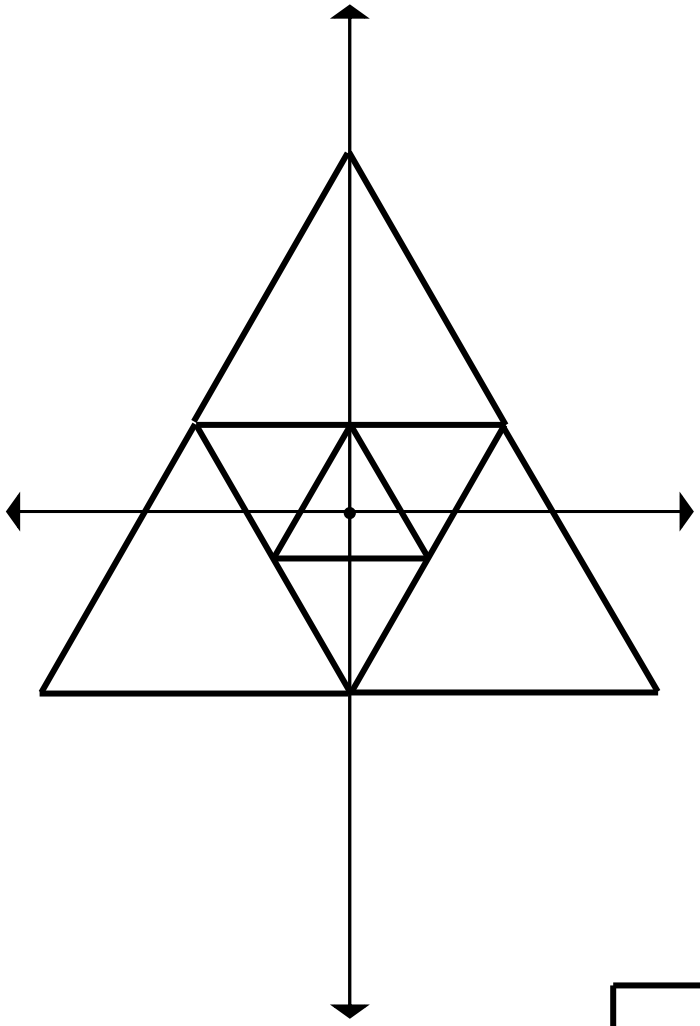
b. Notice that all the side lengths of each of the triangles and squares are the same length AND all the angles are the same measure within the triangles and squares respectively. What is the term that defines this property?

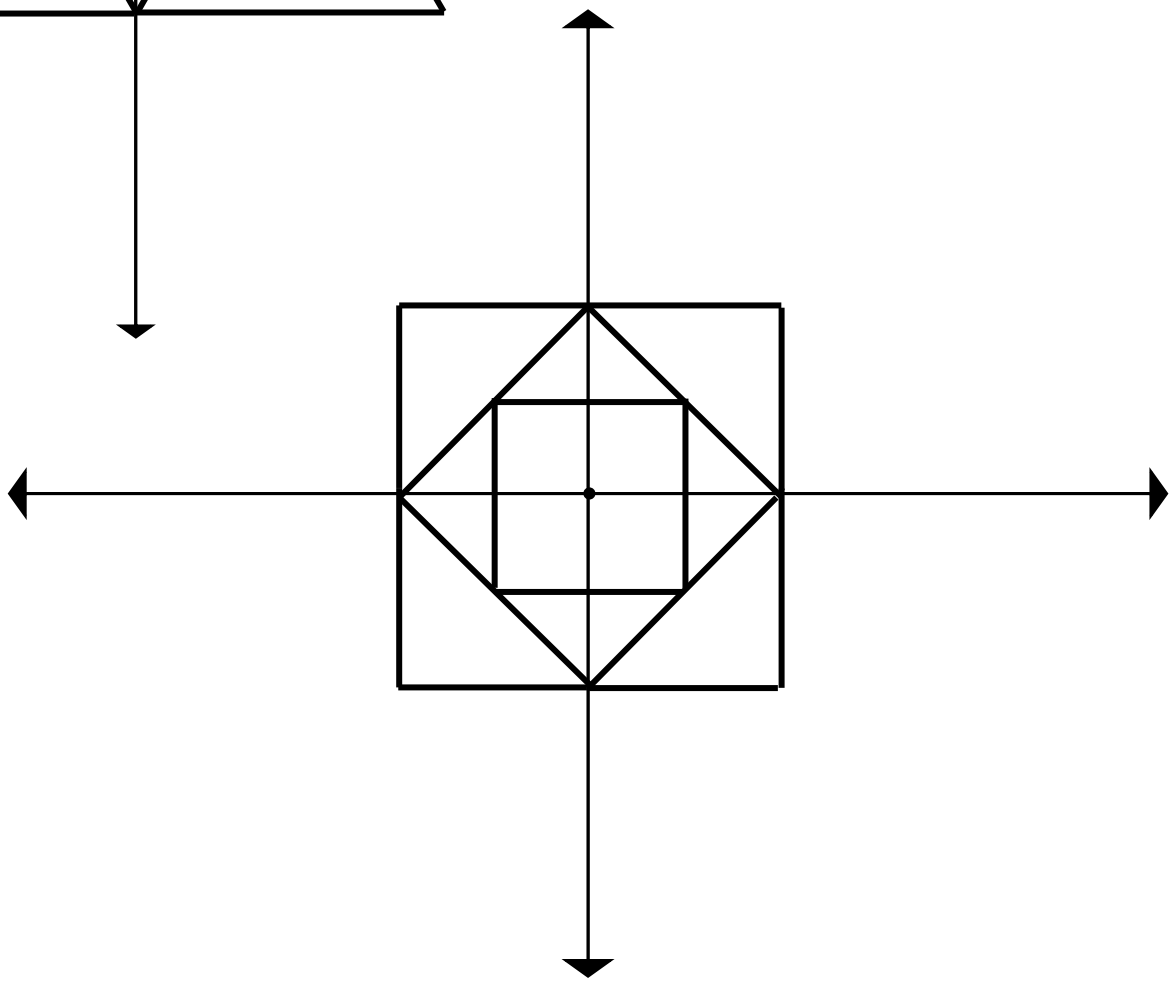
Equilateral + Equiangular = Regular

Why is this property significant?

All regular polygons can be subdivided into regular triangles, making any ensuing calculations, etc. easier to complete.

c. Create a three-layered concentric regular triangle and regular square. Notice: problems 1 and 2 are two-layered concentric triangles and squares, respectively.





Name: _____ Date: _____

# Visionary Illusions

Directions: Everything covered thus far led us to this moment – your practice as an illusionist. Using the theories that we covered, create your own Mandala. The following are the minimum requirements:

1. Minimum of a four-layered concentric shape. You have freedom to choose any equilateral shape less than 7 sides.
2. The size of the inner shape must have area greater than 9 $cm^2$ → i.e. if you use a square, then the side lengths must be longer than 3 cm.
3. The size of the middle and outer layers must be proportional to the size of the inner layer. There are some cases here to pay attention to.
    a. If you choose circles, make sure that the distance between one circle to the next is greater than 3 cm. i.e. if your inner circle has radius 5 cm, then the immediate outer circle must have radius greater than 8 cm.
    b. The layers can be modeled like the examples from the IDLE activity or like our Mandala Madness activity. If you choose that route, you need incorporate the circles for each layer. i.e. if you choose a three-layered concentric triangle or square, every side length must be a diameter of a circle.
    c. If you choose a pentagon of hexagon, make sure that the distance between one layer to the next is greater than 3 cm.
4. Incorporate individual characteristics – create your own designs within the layers via shapes, lines, etc.
5. Use the Four-Color Theorem to color your Mandala. (If you cannot use FCT because of your designs, see how you can adjust your design to ensure the use of FCT)
6. Have fun!

**Rulers, protractors and pencils are necessary for completing the proper measurements and outlines of your Mandalas**

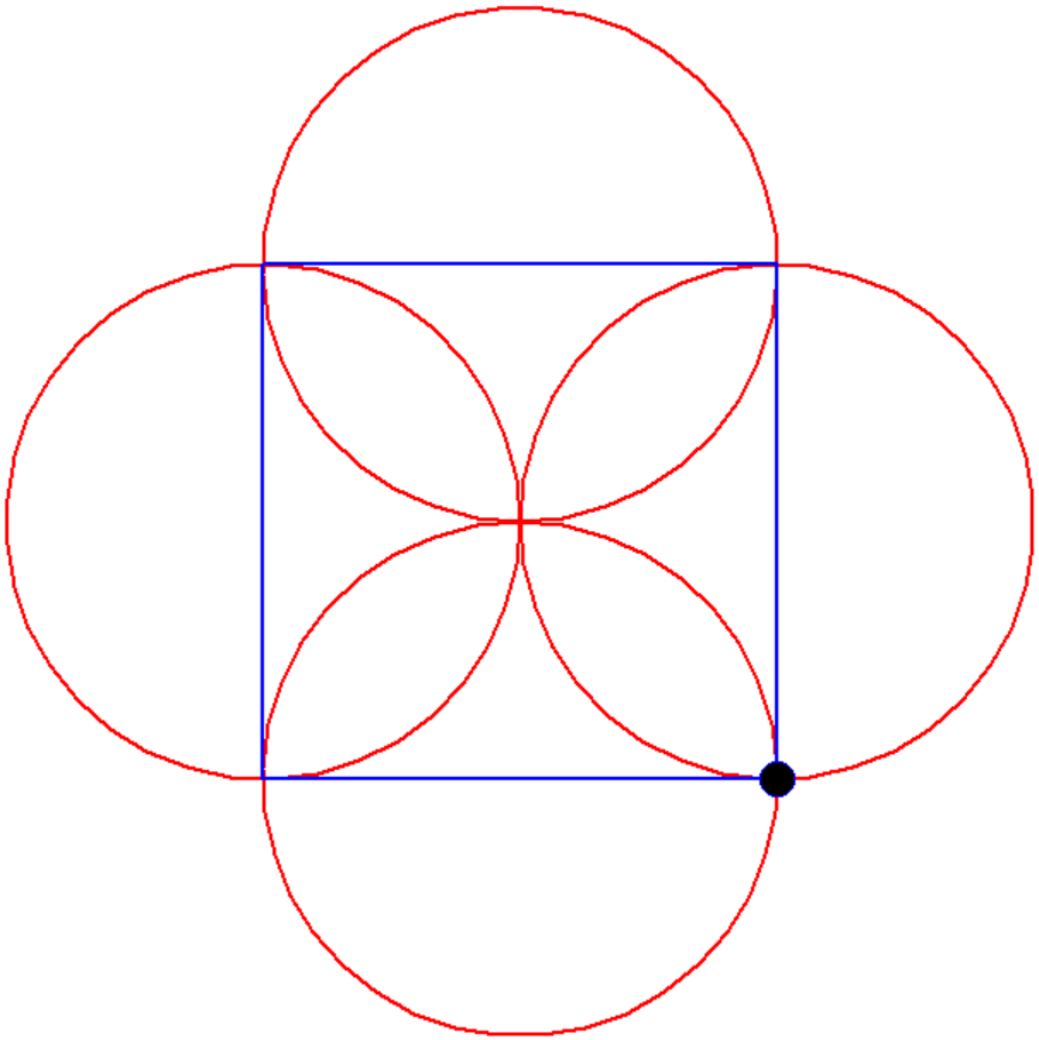
| Rubric for Content Knowledge and Understanding | | | |
|---|---|---|---|
| **Criteria** | **Developing** (does not meet performance standards) | **Proficient** (meets performance standards) | **Advanced** (exceeds performance standards) |
| **Part I: Mandala** | | | |
| ***Completeness and Accuracy of Required Information*** : (1) The shapes and measuerments chosen for the 4-layered concentric Mandala; (2) The colors chosen for application of the Four-Color Theorem. | One or more pieces of required information are omitted | All required information is included | All required information is included |
| | Significant information is incorrect, key terms are used inappropriately, and/or important details are missing | Most significant information is correct, although there are some minor errors or missing details | All information is correct and discussed in detail |
| **Part II: Research** | | | |
| ***Completeness and Accuracy of Required Information*** : (1) What is the Four-Color Theorem; (2) How is it useful, and provide a detailed example; (3) Where else is the Four-Color Theorem seen, and provide a detailed example; (4) How can technology make the creation of Mandalas more accessible? | One or more pieces of required information are omitted | All required information is included | All required information is included |
| | Significant information is incorrect, key terms are used inappropriately, and/or important details are missing | Most significant information is correct, although there are some minor errors or missing details | All information is correct and discussed in detail |
| **Part III: Presentation** | | | |
| ***Completeness and Accuracy of Required Information*** : (1) Detailed explanation of the development of the Mandalas; (2) Detailed explanation of the research content | Less that half of the group members do not present a portion of the group's information | Half of the group members do not present a portion of the group's information | All group members present with equal speaking time |
| | Presentation is shorter than 5 minutes or longer than 25 minutes | Presentation is shorter than 15 minutes or longer than 20 minutes | Presentation is within the 15 to 20 minute time window |
| | Presenters were not able to accurately answer the posed questions after their given presentation | Presenters provided answers to posed questioning with few incorrect assumptions or few misunderstood concepts after their given presentation | Presenters provided knowledgable and coherent answers to posed questioning after their given presentation |

Enrichment

Below is a python code and picture of its output. Students could manipulate the first line (only) to discuss relationships between the various outputs. Recommended range of manipulation: [3,100].

Code:

```
from turtle import *
import math

################################################################################################
##########This is the section that adjusts the following code like a domino effect.#############
################################################################################################

numSides = 4  ##numSides is the number of sides of the inscribed shape: n = 3 >> triangle, etc.

################################################################################################
#############This is the section that draws circles around the central point.##################
################################################################################################

##Variable declarations:

numCircles = numSides           ##numCircles = numSides means that the shape determined by
                                ##numSides is inscribed within a number of circles, which is
                                ##defined as numCircles.

angleCircle = 360/numCircles    ##angleCircle is the calculation of the angle rotation around
                                ##the central point, which is dependent on how many circles are
                                ##drawn around the central point.

##Changes the physical appearance of the output:

shape("circle") #Makes the shape of the pen a circle
pensize(2)      #Integer value that changes the thickness of the line.
speed(15)       #Integer value that changes the speed at which the circle is drawn per second.
size = 150      #Integer value that changes the size of the individual circles drawn around
                #around the central point.

##Loop that draws the circles around thhe central point:

for i in range(numCircles): #Variable within range [1, numCircles].
    rt(angleCircle)         #Turns the pen the exact degree measure for completion of each
                            #circle around the central point to the right.
    pencolor("red")         #Changes the color of the circles drawn to be red.
    circle(size)            #Changes the size of the circle based on the desired size defined.
####################This is the section that draws the inscribed shape.########################
################################################################################################

##Variable declarations:

angleRoto = (180 - angleCircle)/2         ##angleRoto is the angle of rotation about the central point.
                                          ##This angle changes each time that the number of sides of
                                          ##the inscribed shape changes. To calculate this adjustment,
                                          ##we define angleRoto.
initAngle = 180 - angleRoto               ##initAngle is the intitial angle of rotation. Since python's
                                          ##turtle function "leaves" the pen in a particular point on
                                          ##the window after drawing the numCircles, we have to account
                                          ##for this angle adjustment. Hence, we define initAngle.
angle = math.cos(math.radians(angleRoto)) ##angle is the degree measure, converted to radians, that is
                                          ##used in calculations to determining the side length of the
                                          ##inscribed shape.
centerLength = (math.sqrt(2*(size**2)))   ##centerLength is an application of the pythagorean theorem
                                          ##to determine the length of the hypotenuse of a triangle
                                          ##(a subdividision of any overall shape desired).
sideLength = 2 * centerLength * angle     ##sideLength is the overall calculation that determines the
                                          ##exact value of side length of the inscribed shape.
```

```
##Process of drawing the inscribed shape
penup()                 #Lifts the pen so that a line is not drawn to reach a vertex of the
                        #inscribed shape.
rt(angleRoto)           #Turns the pen the exact degree measure for completion of the
                        #inscribed shape to the right.
forward(centerLength)   #Moves the pen forward the exact hypotenuse length.
rt(initAngle)           #Turns the pen the exact degree measure for completion of the
                        #inscribed shape to the right.
pendown()               #Drops the pen so that a line can be drawn for the completion of the
                        #inscribed shape.

##Loop that draws the inscribed shape:

for i in range(numSides): #Variable within range [1, numCircles].
    pencolor("Blue")      #Changes the color of the shape drawn to be blue.
    forward(sideLength)   #Moves the pen forward the exact side length.
    rt(angleCircle)       #Turns the pen the exact degree measure for completion of the
                          #inscribed shape with the common central point.
    speed(5)              #Integer value that changes the speed at which the circle is drawn per second.
```

Output: